Virginia Western Community College
MTH 266
Linear Algebra

Prerequisites
Completion of MTH 263 Calculus I or equivalent with a grade of B or better or MTH 264 Calculus II or equivalent with a grade of C or better.

Course Description
Covers matrices, vector spaces, determinants, solutions of systems of linear equations, basis and dimension, eigenvalues, and eigenvectors. Features instruction for mathematical, physical and engineering science programs.

Semester Credits: 3 **Lecture Hours: 3**

Required Materials
Textbook:
Elementary Linear Algebra with Applications. Hill. 3rd edition. Thomson. ISBN: 9780030103476.

Other Required Materials:
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Course Outcomes

At the completion of this course, the student should be able to:

- Matrices and Systems of Equations
  - Use correct matrix terminology to describes various types and features of matrices (triangular, symmetric, row echelon form, et.al.)
  - Use Gauss-Jordan elimination to transform a matrix into reduced row echelon form
  - Determine conditions such that a given system of equations will have no solution, exactly one solution, or infinitely many solutions
  - Write the solution set for a system of linear equations by interpreting the reduced row echelon form of the augmented matrix, including expressing infinitely many solutions in terms of free parameters
  - Write and solve a system of equations modeling real world situations such as electric circuits or traffic flow
- Matrix Operations and Matrix Inverses
  - Perform the operations of matrix-matrix addition, scalar-matrix multiplication, and matrix-matrix multiplication on real and complex valued matrices
  - State and prove the algebraic properties of matrix operations
  - Find the transpose of a real valued matrix and the conjugate transpose of a complex valued matrix
  - Identify if a matrix is symmetric (real valued)
  - Find the inverse of a matrix, if it exists, and know conditions for invertibility.

- Use inverses to solve a linear system of equations
- Determinants
  - Compute the determinant of a square matrix using cofactor expansion
  - State, prove, and apply determinant properties, including determinant of a product, inverse, transpose, and diagonal matrix
  - Use the determinant to determine whether a matrix is singular or nonsingular
  - Use the determinant of a coefficient matrix to determine whether a system of equations has a unique solution
- Norm, Inner Product, and Vector Spaces
  - Perform operations (addition, scalar multiplication, dot product) on vectors in Rn and interpret in terms of the underlying geometry
  - Determine whether a given set with defined operations is a vector space
- Basis, Dimension, and Subspaces
  - Determine whether a vector is a linear combination of a given set; express a vector as a linear combination of a given set of vectors
  - Determine whether a set of vectors is linearly dependent or independent
  - Determine bases for and dimension of vector spaces/subspaces and give the dimension of the space
  - Prove or disprove that a given subset is a subspace of Rn
  - Reduce a spanning set of vectors to a basis
  - Extend a linearly independent set of vectors to a basis
  - Find a basis for the column space or row space and the rank of a matrix
  - Make determinations concerning independence, spanning, basis, dimension, orthogonality and orthonormality with regards to vector spaces
- Linear Transformations
  - Use matrix transformations to perform rotations, reflections, and dilations in Rn
  - Verify whether a transformation is linear
  - Perform operations on linear transformations including sum, difference and composition
  - Identify whether a linear transformation is one-to-one and/or onto and whether it has an inverse
  - Find the matrix corresponding to a given linear transformation T: Rn -> Rm
  - Find the kernel and range of a linear transformation
  - State and apply the rank-nullity theorem
  - Compute the change of basis matrix needed to express a given vector as the coordinate vector with respect to a given basis
- Eigenvalues and Eigenvectors
  - Calculate the eigenvalues of a square matrix, including complex eigenvalues.
  - Calculate the eigenvectors that correspond to a given eigenvalue, including complex eigenvalues and eigenvectors.
  - Compute singular values
  - Determine if a matrix is diagonalizable
  - Diagonalize a matrix

Major Topics to be Included

- Matrices and Systems of Equations
- Matrix Operations and Matrix Inverses
- Determinants
- Norm, Inner Product, and Vector Spaces
- Basis, Dimension, and Subspaces
- Linear Transformations
- Eigenvalues and Eigenvectors

Topical Description

1	Introduction to Linear Equations and Matrices
1.1	Introduction to Linear Systems and Matrices
1.2	Gaussian Elimination
1.3	The Algebra of Matrices: Four Descriptions of the Product
1.4	Inverse and Elementary Matrices
1.5	Gaussian Elimination as a Matric Factorization
1.6	Transposes, Symmetry, and Band Matrices; an Application
2	Determinants
2.1	The Determinant Function
2.2	Properties of Determinants
2.3	Finding detA Using Signed Elementary Products
2.4	Cofactor Expansion; Cramer's Rule
3	Vector Spaces
3.1	Vectors in 2 and 3 Spaces
3.2	Euclidean n-space
3.3	General Vector Spaces
3.4	Subspaces, Span, Null Spaces
4	Linear Trans., Orthogonal Projections, and Least Squares
4.1	Matrices as Linear Transformations
4.2	Relationships Involving Inner Products
4.3	Least Squares and Orthogonal Projections
4.4	Orthogonal Bases and the Gram-Schmidt Process
4.5	Orthogonal Matrices, QR Decompositions, and Least Squares
5	Eigenvectors and Eigenvalues
5.1	A Brief Introduction to Determinants
5.2	Eigenvalues and Eigenvectors
5.3	Diagonalization
5.4	Symmetric Matrices

Notes to Instructors

None.

ADA Statement (PDF)
Title IX Statement (PDF)